Ultrashort relativistic electromagnetic solitons

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Ultrashort high-intensity electromagnetic solitons in both underdense and overdense plasmas are investigated. Comparison is made for solitons with smooth and sharp electron density profiles. It is found that subcycle relativistic solitons can propagate from low-density to high-density plasmas.

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I. INTRODUCTION

Ultraintense light pulses can propagate in plasmas as solitons for very long distances without apparent change in property, as have been observed in particle-in-cell (PIC) simulations [1–4]. Recent rapid development in laser technology makes it possible to produce relativistic light pulses of extremely short duration, say with only a few laser periods. Subcycle ultrashort relativistic solitons are of interest in the study of ultrafast phenomena and have been found in a three-dimensional (3D) PIC simulation [5].

Existing analytical studies on relativistic solitons are usually based on one-dimensional theories [6], and several novel solutions have been found [7–12]. In most studies, the electron density profile is assumed to be smooth, or well behaved. Under this ansatz, stationary subcycle relativistic solitons were shown to exist for plasma densities between n_c and $1.5n_c$ [9]. Propagating subcycle solitons seem to be possible only at low intensities ($a \ll 1$, where $a = eA/mc^2$ is the normalized vector potential) with weak density response [11]. The analytical theory also shows that for given pulse width and plasma density only one soliton solution of precisely given field energy and group velocity is possible [11]. Furthermore, for the same pulse width one finds that high plasma density corresponds to high field intensity. As a result, such a soliton cannot easily propagate in the plasma, especially in the plasma produced by laser solid interaction, because the plasma density is usually not homogeneous.

On the other hand, with the extremely high ponderomotive pressure of a short intense light pulse, the electron density need not be smooth since the plasma response time is too long for boundary smoothing to occur. In this paper, we allow the electron density profile to be (self-consistently) sharp. That is, the profile can contain infinite derivatives. It is found that solitons of subcycle duration can exist. Furthermore, for given plasma density and pulse duration, there exists an infinite number of solutions, and higher field energy corresponds to higher group velocity of the soliton. A soliton with sharp density profile can easily propagate in the direction of increasing plasma density, with the group velocity becoming gradually smaller. Therefore, an ultrashort relativistic soliton may penetrate into the overdense plasma of high density.

II. FORMULATION

To investigate the propagation of relativistic solitons with sharp density profiles, we assume that the electrons obey the relativistic cold-fluid equations and the ions are stationary. The electron momentum and continuity equations are then

$$(\partial_t + u\partial_z)(\gamma u) = \partial_z \phi - \frac{1}{2\gamma}\partial_z a^2, \tag{1}$$

$$\partial_t n_e + \partial_z(n_e u) = 0, \tag{2}$$

where t, z, u, n_e, and ϕ are the normalized time and space, electron velocity and density, and scalar potential, respectively, $\gamma = [(1+a^2)/(1-u^2)]^{1/2}$ is the relativistic factor, and u is the lingitudinal velocity. Here, the normalizations $\omega_L t$, $\omega_L z/c$, u/c, n_e/n_c, and $e\phi/mc^2$ are used, and n_c is the critical density for a laser of frequency ω_L. Note that the time is normalized using the laser frequency ω_L instead of the plasma frequency ω_{pe}.

Under the quasistatic approximation [11,13], it is convenient to use the independent variables $\xi = z - v_g t$ and $\tau = t$, where v_g is the normalized group velocity of the soliton. We obtain from Eqs. (1) and (2)

$$\gamma(1 - uv_g) - \phi = 1, \tag{3}$$

$$n_e(v_g - u) = n_i v_g, \tag{4}$$

and Poisson's equation can be written as

$$\partial_\xi^2 \phi = n_e - n_i = \frac{u}{v_g - u} n_i. \tag{5}$$

For a circularly polarized laser pulse, the vector potential of the electromagnetic field can be represented by $A = a(\xi)\exp\{i[\omega\tau + \theta(\xi)]\}(\hat{x} + i\hat{y})$. Unlike in studies of laser pulse propagation, for subcycle soliton investigation we do not assume that there already exists a propagating light wave and consider the modulational evolution of its envelope. Instead, the interaction of the entire electromagnetic wave field with electrons is considered. From the wave equation, one can obtain two constants of integration,

$$M = a^2[(1 - v_g^2)\theta' + \omega v_g], \tag{6}$$

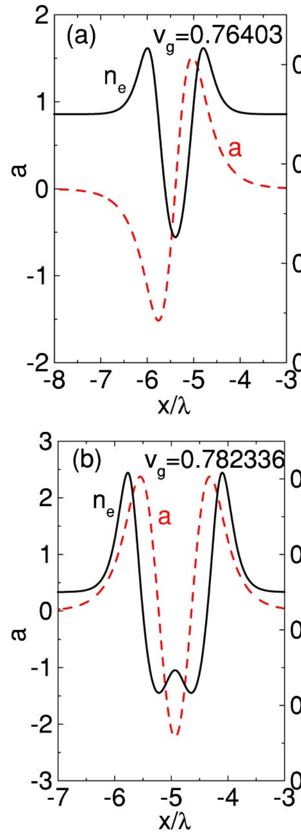

FIG. 1. Shortest relativistic solitons with smooth electron density profile in an underdense plasma of background density $0.5n_c$.

$$W = \frac{a'^2}{2} - \frac{\phi'^2 + 2n_i\phi}{2(1-v_g^2)} + \frac{a^2}{2} + \frac{M^2}{2a^2(1-v_g^2)^2} - \frac{n_i\gamma v_g u}{(1-v_g^2)}, \tag{7}$$

where the prime denotes derivative with respect to the argument. For an infinite plasma, we have $M=W=0$. Therefore,

$$\theta = -\frac{\omega v_g}{1-v_g^2}\xi + \theta_0, \tag{8}$$

where θ_0 is a constant. Thus, the frequency of the laser is $\omega_L = \omega/(1-v_g^2)$.

For considering sharp electron density boundaries, it is convenient to separate the plasma into three regions $-\infty < \xi < -\xi_m/2$, $-\xi_m/2 < \xi < \xi_m/2$, and $\xi_m/2 < \xi < \infty$, with only the center one not containing electrons. That is, ξ_m is the width of the soliton. In the electron-free regions (where there are only stationary ions), we have $A = a_1\exp(i\tau - i\xi) + a_2\exp(i\tau + i\xi + i\theta_2)$, which satisfies the source-free wave equation. Thus, invoking continuity of the transverse electric and magnetic fields at the soliton boundaries, one finds [8]

$$a_1^2 - a_2^2 = \frac{v_g}{1-v_g^2}a^2, \tag{9}$$

$$\left(1 - \frac{v_g}{1-v_g^2}\right)^2 a^2 + a'^2 = 4a_2^2, \tag{10}$$

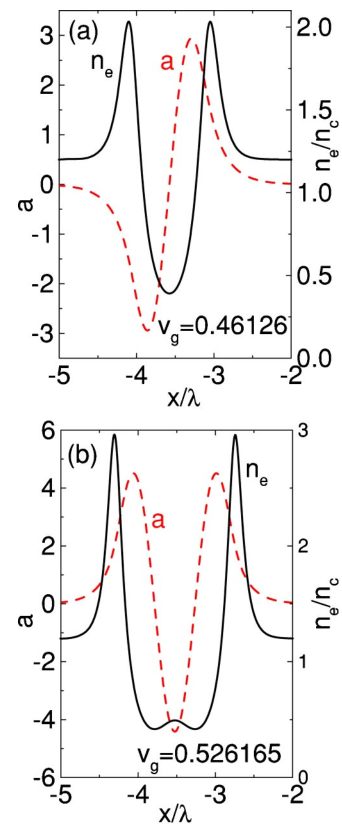

FIG. 2. Shortest relativistic solitons with smooth electron density profile in an overdense plasma of background density $1.2n_c$.

$$\cos(\theta_2 - \xi_m) = (a^2 - a_1^2 - a_2^2)/(2a_1a_2), \tag{11}$$

at the right soliton boundary. There are similar results at the left soliton boundary. The normalized electrostatic field at the right surface is then $E_z = -n_i\xi_m/2$. For a subcycle soliton, we have $\theta_2 = 0$.

III. SMOOTH SOLITONS

First, we shall consider solitons with a smooth density profile. For an infinite underdense plasma of density $n = 0.5n_c$, the two shortest solitons are illustrated in Fig. 1. For underdense plasmas, we have $\phi = (1+a^2)^{1/2} - 1$ when $a \to 0$. Therefore, from Eq. (7) we have $v_g \geqslant (1-n_i)^{1/2}$. There are subcycle solitons ($p=0$, where p is the number of zeros of the a profile) only in the limit of weak density response, and they are not relativistic.

For overdense plasmas with density between n_c and $1.5n_c$, it is known that there are solitons with zero group velocity, as was shown by Esirkepov *et al.* [9] for $n_i = 1.5n_c$. In Fig. 2, we show two *propagating* solitons of the shortest width for $n_i = 1.2n_c$. Such solitons are associated with precisely defined group velocity and field energy, so that there are no neighboring solutions. As in underdense plasmas, the shortest relativistic soliton still has one wave cycle.

For an overdense plasma of density larger than $1.5n_c$, no subcycle ($p=0$) soliton solution exists. Furthermore, there is also no other short soliton. For example, for a plasma of

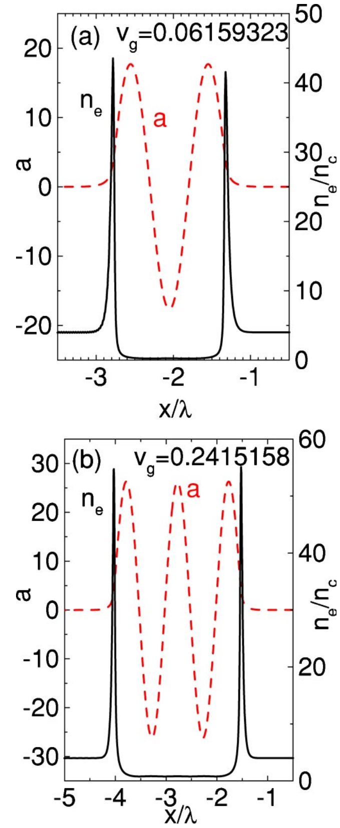

FIG. 3. Two relativistic solitons of smooth electron density profile in an overdense plasma of background density $4n_c$; (a) $p=2$ and (b) $p=4$.

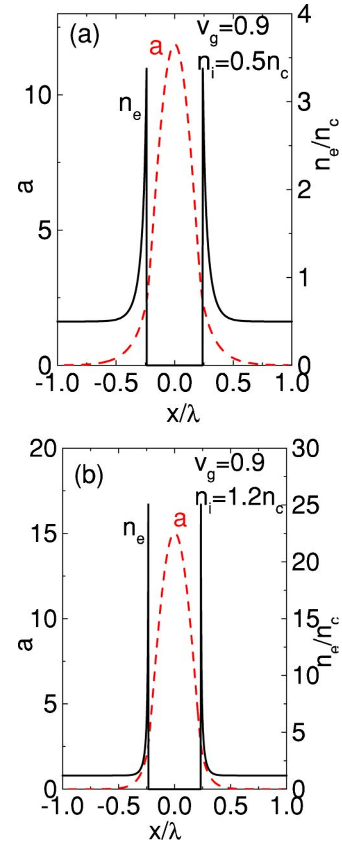

FIG. 4. Subcycle relativistic solitons of group velocity $v_g=0.9$ with sharp electron density profiles in overdense plasmas of background densities (a) $0.5n_c$ and (b) $1.2n_c$.

density $4n_c$, the shortest soliton has $p=2$, as illustrated in Fig. 3 together with the soliton with $p=4$. These solitons also require exactly defined (for a given background plasma density) group velocity and electromagnetic field energy. Furthermore, for given p, denser plasma requires larger field energy. Thus, creating such solitons in reality is very difficult. It is also impossible for such a soliton to propagate up a density gradient since it cannot gain energy as it moves.

IV. SHARP SOLITONS

In real situations involving ultrashort ultraintense laser pulses, the local electrons can be completely expelled by the relativistic ponderomotive force. The expelled electrons, being balanced by the space-charge and ponderomotive fields, pile up just outside the intense field region, resulting in an abrupt and steep jump in the density profile (in the cold-fluid representation). For this case, we found that for soliton propagation in an underdense plasma, the group velocity v_g of the soliton must still be larger than $(1-n_i)^{1/2}$, as for the smooth solitons. However, for given plasma, density, subcycle solitons can now exist if the field energy is sufficiently large. Furthermore, larger field energy now corresponds to larger soliton group velocity. In Fig. 4(a), a subcycle soliton of group velocity $v_g=0.9$ in a plasma of density $0.5n_c$ is shown.

In overdense plasmas, solitons with zero group velocity are possible. For background plasma density between n_c and $1.5n_c$, we found subcycle solitons of zero group velocity only with smooth electron density profile, as discussed above. For plasmas with background density larger than $1.5n_c$, subcycle solitons with sharp electron density profile and zero group velocity exist. An example of such a stationary structure was found elsewhere [14] for $n_i=4n_c$ in a completely different context, and the corresponding maximum field amplitude is $a\sim 6$. Figure 4(b) shows a *propagating* soliton of group velocity $v_g=0.9$ in an overdense plasma with $n_i=1.2n_c$. The maximum field amplitude here is $a\sim 15$. Our results also show that in order for a soliton to propagate with the same group velocity, a subcycle soliton in plasma of larger density must have larger field energy. Thus, when a subcycle soliton propagates up a density gradient, it moves slower and slower until the plasma density becomes too large. Depending on the instantaneous values of its parameters, it can then either stop and stay there, or get reflected.

V. DISCUSSION

In this paper, we have shown that solitons with sharp electron density profiles can exist in group-velocity and density regimes where smooth solitons are forbidden. That is, the specific soliton type can only be determined by the conditions under which they are initially created by laser pulses, since it is difficult for pulses of one type to evolve self-consistently into another, especially when the existence regime of some pulse types is rather singular.

We note that the type of solitons discussed here is unique to ultraintense ultrashort pulse light, since for weaker pulses on longer time scales, trapped-particle and thermalization effects can become important, and surface-current effects as well as ion dynamics can also play a role [10]. In this case, the propagation characteristics of the different types of solitons will certainly change. On the other hand, in applications such as production of and light scattering from attosecond electron bunches in plasma channels, a long lifetime for such ultrarelativistic ultrashort light pulses is not required. Short laser pulses containing only a few laser periods but having kJ energy can be employed for fast ignition [15] in inertial fusion experiment since the ultrashort solitons with sharp electron density profiles can easily penetrate deeper into the burn core than the traditional picosecond laser pulses.

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